

ঢাকা বিশ্ববিদ্যালয় অধিভুক্ত কলেজ

বিএসসি (সম্মান) ১ম বর্ষ পরীক্ষা-২০২০

বিষয় : গণিত \triangleright কোর্স শিরোনাম : Calculus-I

কোর্স কোড : MAT-102

সময় : ১ ঘণ্টা ৫০ মিনিট

পূর্ণমান : ৭০

N. B. : Answer any 05 (Five) out of the following 08 (Eight) questions.

Figures given in the right margin indicate the marks of the respective questions

1. (a) Define domain and range of a function. Find the domain and range of the following functions:

(i) $f(x) = \sqrt{x^2 - 5x + 6}$ (ii) $g(x) = \frac{x}{(x-1)(x-2)}$ 10

(b) Sketch the graph of the function $f(x) = e^{-x^2}$. 42. Discuss the continuity and differentiability at $x = 0$ and $x = \frac{\pi}{2}$ of the function

$$f(x) = \begin{cases} 1 & \text{when } x < 0 \\ 1 + \sin x & \text{when } 0 \leq x < \frac{\pi}{2} \\ 2 + \left(x - \frac{\pi}{2}\right)^2 & \text{when } x \geq \frac{\pi}{2} \end{cases}$$
 14

3. (a) Find $\frac{dy}{dx}$ if $y = \tan x^{\cot x} + \cot x^{\tan x}$ 7(b) If $\ln y = \tan^{-1} x$, then show that, $(1 + x^2) y_{n+2} + (2nx + 2x - 1) y_{n+1} + n(n+1) y_n = 0$. 7

4. (a) State and prove Rolle's theorem. 7

(b) State Mean value theorem and verify the Mean value theorem for the function $f(x) = 3 + 2x - x^2$ in the interval $(0, 1)$. 75. Evaluate the following Integrals : (i) $\int \frac{1 - \sin x - \cos x}{1 + \sin x - \cos x} dx$

(ii) $\int \frac{dx}{(x^2 + 1)\sqrt{x^2 + 4}}$

(iii) $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ 14

6. (a) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n \pi dx$, prove that, $I_n + I_{n-2} = \frac{1}{n-1}$. 7

(b) Evaluate $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{1}{\sqrt{n^2 - 1^2}} + \frac{1}{\sqrt{n^2 - 2^2}} + \dots + \frac{1}{\sqrt{n^2 - (n-1)^2}} \right]$ 7

7. (a) Define Gamma function and Beta function. Show that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$ 10

(b) Show that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ 4

8. (a) Define area. Find the area of a loop of the curve $a^2 y^2 = x^2 (a^2 - x^2)$ 8(b) Find the perimeter of the cardioid $r = 2(1 - \cos \theta)$ 6