Complex Analysis Short Suggestion 2023

Honours 3rd Year

১. sinhz = i সমীকরণের সকল সমাধান নির্ণয় কর। [NUH-2017]

২. coshz = 2 সমীকরণের সকল সমাধান নির্ণয় কর। [NUH-2010,12,18,20]

৩. যে কোন জটিল সংখ্যা $Z_1, Z_2, \ldots, Z_n$ এর জন্য প্রমাণ কর যে,

(i) $|z_1 + z_2| \le |z_1| + |z_2|$ [NUH-2010,12,19]

(ii) $|z_1 + z_2| \le |z_1| + |z_2|$ [NUH-2017,19]

৪. দুইটি জটিল সংখ্যা বাহির কর যাদের যোগফল 4 এবং গুণফল ৪ । [NUH-2012,16]

৫. নিচের এলাকা গুলোকে জ্যামিতিক ভাবে বর্ণনা কর।

(i) $|z-i|=|z+i|$ [NUH-2010,12,16]

(ii) $Re\left(\frac{1}{z}\right) < \frac{1}{2}$ [NUH-2010,15,18,21]

(iii) $|z+1+i|=|z-1+i|$ [NUH-2020]

৬. Necessary conditions for f(z) to be analytic.

The necessary condition for w = f(z) = u(x, y) + iv(x, y) to be analytic at a point z = x + iy of its domain is that the four partial $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ *should exist and satisfy the Cauchy-Riemann partial differential equations* $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$. [NUH-2010,12,16,18,19,20]

৭. Sufficient condition for f(z) to be analytic .

The function w = f(z) = u(x, y) + iv(x, y) is analytic in a domain D if

(i) u, v are differentiable in D and $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$ (ii) The partial derivatives $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ are all continuous in D. [NUH-2013,17,21]

৮. Prove that f(z) = In z has a branch point at z = 0. [NUH-2017,20]

৯. Let $f(z) = u + iv = \frac{x^3-3xy^3+i(y^3-3x^2y)}{x^2+y^2}$, when $z\neq0$ and f(z) = 0 when z = 0 Show that f(z) is continuous and the Cauchy-Riemann equations are satisfied but f(z) is not differentiable at z = 0 . [NUH-2013,19,20]

১০. Show that the function $f(z)=u+iv = \frac{(1+i)x^3-(1-i)y^3}{x^2+y^2}$ if $z\neq0$ and f(0) = 0 if z = 0 ,is continuous and that the Cauchy-Riemann equations ar satisfied at the Origin, yet f'(0) does not exist. [NUH-2011,17,21]

১১. *Prove that f(z) = $|z|^2$ is continuous every where but not differentiable except at the origin.* [NUH-2015,19,20]

১২. *Show that an analytic function with constant modulas is constant.* [NUH-2011,15,18,20]

১৩. *Show that u = $3x^2y + 2x^2 - y^3 - 2y^2$ is a harmonic fuction and hence find its harmonic conjugate v is f(z) = u + iv is analytic.* [NUH-2016,18,20]

১৪. *Find the harmaonic conjugate of u = 2x(1 - y).* [NUH-2014,21]

১৫. *If a function f(z) is continuous on a contour C of length L, and if M be the upper bound of | f(z) | on C then* $\left|\int_c f(z)\,dz\right| \le ML$ [NUH-2011,14,18]

১৬. *If f(z) is analytic in a region R and on its clos boundary C with derivative f'(z) which is continuous at all poir inside R and on C, then* $\oint_c f(z)dz = 0$. [NUH-2011,13,15,16]

১৭. *Let f(z) be analytic inside and on a simple close curve C. If a is any point inside C, then*

$$f(a)=\frac{1}{2\pi i}\oint_c \frac{f(z)}{z-a}\,dz$$ [NUH-2014,16,18,20]

১৮. *Let f(z) be analytic inside and on a simple close curve C and a is a point inside C. Then*

$$f'(a)=\frac{1}{2\pi i}\oint_c \frac{f(z)}{(z-a)^2}\,dz$$ [NUH-2010,12,15,17,21]

১৯. *Show that,* $\oint_c \frac{e^{3z}}{z-\pi i}\,dz = \begin{cases} -2\pi i, \text{ if } C \text{ is the circle } |z-1| = 4 \\ 0, \text{ if } C \text{ is the ellipse } |z-2|+|z+2| = 6 \end{cases}$ [NUH-2016,20]

২০. *Show that,* $\oint_c \frac{e^{tz}}{z^2+1}\,dz = 2\pi i \sin t$ *where C is the circle |z|= 3and t > 0.* [NUH-2011,13,18,20]

২১. *What is Cauchy's integral formula? Using this evaluate* $\int_c \frac{z\,dz}{(9-z^2)(z+i)}$ *, where C is the circle |z| =2 describe in the positive sense.* [NUH-2013,20]

২২. *If f(z) is analytic for all values of z insid circle C with centre at a, then*

$$f(z)= f(a) + (z-a)\, f'(a) + \frac{(z-a)^2}{\lfloor 2} f''(a) + \frac{(z-a)^3}{\lfloor 3} f''(a) + \ldots$$ [NUH-2013,16,21]

২৩. *কসির অবশেষ উপপাদ্য বর্ণনা ও প্রমাণ কর।* [NUH-2021]

২৪. *রচির উপপাদ্য বর্ণনা দাও ও প্রমাণ কর।* [NUH-2012,14,15,17,21]

২৫. *ব্যতিক্রমী বিন্দুগুলির শ্রেণীবিভাগ কর। প্রতিটি ধরণের একটি করে উদাহরণ দাও।* [NUH-2011,15,17]

২৬. $f(z) = \frac{\sin\left(\frac{1}{z}\right)}{(z^2-1)^2}$ *এর সিংগুলারিটি নির্ণয় কর।* [NUH-2020]

২৭. $f(z)=\sin z$ ফাংশনকে $z=\frac{\pi}{4}$ বিন্দুর প্রতিবেশে টেইলর ধারায় প্রকাশ কর। [NUH-2010,16,18]

২৮. Expand the function f(z)= i laurent series for the following region:

(i) $1<|z|<3$ [NUH-2020] (ii) $|z|>3$, (iii) $0<|z+1|<2$, (iv) $|z|<1$ [NUH-2017]

২৯. Find the Laurent expansion of the function $f(z)=\frac{z^2+1}{(z+1)(z-2)}$ in each of the regions:

(i) $1<|z|<1$ [NUH-2012,18,20]
(ii) $0<|z|<1$ [NUH-2012,18]

৩০. If the mapping w = f(z) is conformal, then f(z) an analytic function of z. [NUH-2012,15,21]

৩১. Find a bilinear transformation which transforms points $z = 0, -1, -1$ into $w = i, 1, 0$ respectively.

[NUH-2017]

৩২. Find a bilinear transformation which map the points $i, -i, 1$ of the z-plane into the points $0, 1, -\infty$ of w-plane respectively. [NUH-2012,20]

৩৩. Show that the transformation $w=\frac{2z+3}{z-4}$ ranges the circle $x^2+y^2-4x=0$ into the stright line $4u+3=0$. Explain why the curve obtained is not a circle. [NUH-2014,17]

৩৪. Show that the transformation $w=\frac{1+iz}{z+i}$ maps the real axis of the z-plane onto a circle in the w-plane. Find its centre and radius. [NUH-2010, 16, 18]

৩৫. Find a bilinear transformation which transform the unit circle (disc $|z|\le 1$ into the unit circle $|w|\le 1$.

[NUH-2014,18,20]

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